BATU-EXAM

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Engg Maths 2 Department

U.4.1

MET Bhujbal Knowledge City
Institute of Technology - Polytechnic - Diploma/B.Tech
CLASS TEST - I / II / Mid Sem Exam Date :

Name : Roll No :
Course Title : Class : Sem :
Division :

Main Answer book	Supplement	Total	Question No.	1						2				3				Total Marks
			Max. Marks															
				a	b	c	d	e	f	a	b	c	d	a	b	c	d	
Supervisor's Sign.			Marks Obtained															

Fourier Series

If $f(x)$ is defined and periodic in the interval $[a,b]$, then its fourier series is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi}{b-a}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi}{b-a}\right)$$

By Euler's formula,

$$a_0 = \frac{2}{b-a}\int_a^b f(x)\,dx$$

$$a_n = \frac{2}{b-a}\int_a^b f(x)\cdot\cos\left(\frac{2n\pi x}{b-a}\right)dx$$

$$b_n = \frac{2}{b-a}\int_a^b f(x)\cdot\sin\left(\frac{2n\pi x}{b-a}\right)dx$$

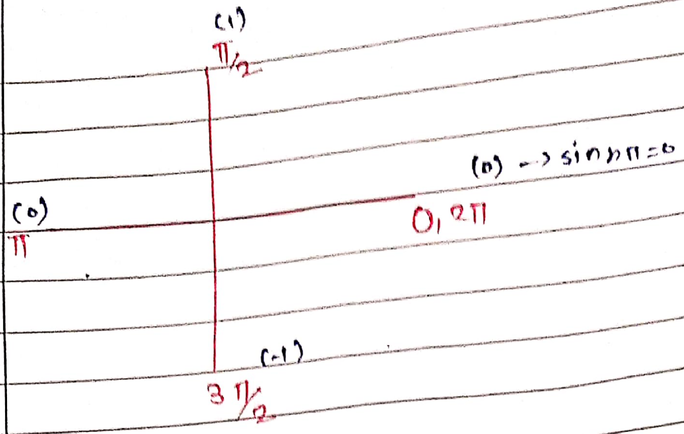
For $b-a = 2\pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where, $a_0 = \frac{1}{\pi}\int_a^b f(x)\,dx$

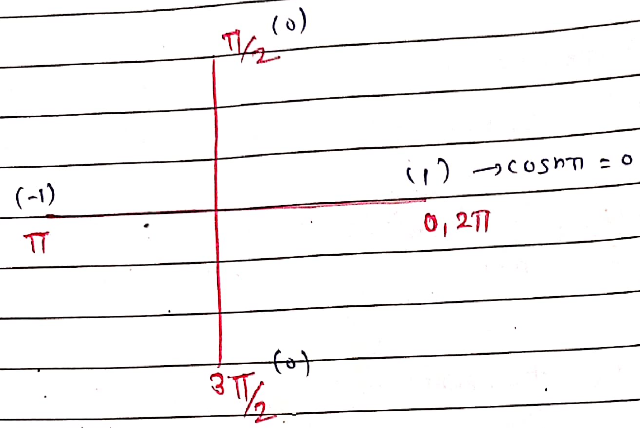
$$a_n = \frac{1}{\pi}\int_a^b f(x)\cdot\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_a^b f(x)\cdot\sin nx\,dx$$



$\sin n\pi = 0$

$\sin 2n\pi = 0$



$\cos n\pi = (-1)^n$

$\cos 2n\pi = 1$

$\cos(2n+1)\pi = -1$

$\cos(2n-1)\pi = -1$

E.x. Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, \quad (-\pi < x < \pi)$$

→ Here, $f(x) = x^2$ and $b-a = 2\pi$

∴ The fourier series for $f(x) = x^2$ is,

$$x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{——— } (1)$$

Now,

$$a_0 = \frac{1}{\pi} \int_a^b f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx$$

Div:-C Present 26/05/23 10 to 11 | 2, 6, 8, 9, 11, 12, 14, 15, 16, 21, 22, 24, 25, 27, 28, 29, 31, 32, 33, 34, 36, 37, 38, 40, 41, 43, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 68, 20,

$$= \frac{1}{\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{1}{3\pi}\left[\pi^3 - (-\pi)^3\right] = \frac{2\pi^3}{3\pi} = \frac{2\pi^2}{3}$$

$$\therefore \boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_a^b f(x)\cos nx \, dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \underset{I}{x^2} \underset{II}{\cos nx} \, dx$$

$$= \frac{1}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - (2x)\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

chain Rule of integration biparts

$$= \frac{1}{\pi}\left[\frac{x^2 \sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2\sin nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\pi^2 \sin n\pi}{n}} - \cancel{\frac{(-\pi)^2 \sin n(-\pi)}{n}} + \right.$$

$$\frac{2\pi\cos n\pi}{n^2} - \frac{2(-\pi)\cos n(-\pi)}{n^2} -$$

$$\left. \cancel{\frac{2\sin n\pi}{n^3}} + \cancel{\frac{2\sin n(-\pi)}{n^3}}\right]$$

$$= \frac{1}{\pi}\left[\frac{2\pi\cos n\pi}{n^2} + \frac{2\pi \cdot \cos n\pi}{n^2}\right]$$

$$= \frac{1}{\pi} \quad \frac{4\pi\cos n\pi}{n^2} = \frac{4(-1)^n}{n^2}$$

$$a_n = \frac{4(-1)^n}{n^2} \qquad \because \sin n\pi = 0, \quad \cos n\pi = (-1)^n$$

Div:-B 24/05/23 Present 1:30 to 2:30 | 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 28, 29, 30, 31, 32, 33, 34, 36, 39, 41, 42

Div:-B 25/05/23 Present 8 to 9 | 1, 3, 4, 5, 6, 7, 8, 10, 13, 14, 15, 17, 18, 20, 21, 22, 23, 28, 31, 32, 34, 39, 41, 42

Div:-A
Present
25/05/23
12:30 to 1:30

2,3,4,5,6,7,8,9,10,
27,28,

$$b_n = \frac{1}{\pi}\int_a^b f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cdot \sin nx\,dx$$

$$= \frac{1}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) - (2x)\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{x^2 \cdot \cos nx}{n} + \frac{2x \cdot \sin nx}{n^2} + \frac{2\cos nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\pi^2 \cdot \cos n\pi}{n} + \frac{(-\pi)^2 \cdot \cos n(-\pi)}{n} + \frac{2\pi \sin n\pi}{n^2} - \frac{2(-\pi)\sin n(-\pi)}{n^2} + \frac{2\cos n\pi}{n^3} - \frac{2\cos n(-\pi)}{n^3}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi^2 \cos n\pi}{n} + \frac{\pi^2 \cos n\pi}{n} + \frac{2\cos n\pi}{n^3} - \frac{2\cos n\pi}{n^3}\right]$$

$$\boxed{b_n = 0}$$

$\therefore$ eq^n (1), becomes,

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$$

E.x. Obtain the fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

$\rightarrow$ Here,

$f(x) = e^{-x}$ and $b - a = 2\pi$

$\therefore$ The fourier series for $f(x) = e^{-x}$ is

$$e^{-x} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{——— (1)}$$

Now,

$$a_0 = \frac{1}{\pi}\int_a^b f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\,dx = \frac{1}{\pi}\left[\frac{e^{-x}}{-1}\right]_0^{2\pi}$$

Poesent 26|05|23 12:30 to 1:30 | 34, 39,

$$= \frac{-1}{\pi}\left[e^{-2\pi} - e^{0}\right] = \frac{1-e^{-2\pi}}{\pi}$$

$$\therefore \boxed{a_0 = \frac{1-e^{-2\pi}}{\pi}}$$

$$a_n = \frac{1}{\pi}\int_a^b f(x).\cos nx\,dx = \frac{1}{\pi}\int_0^{2\pi} e^{-x}.\cos nx\,dx$$

Use formula,

$$\int e^{ax}.\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{(1+n^2)}(-\cos nx + n\sin nx)\right]_0^{2\pi}$$

$$= \frac{1}{\pi(1+n^2)}\left[n e^{-x}\sin nx - e^{-x}\cos nx\right]_0^{2\pi}$$

$$= \frac{1}{\pi(1+n^2)}\left[(0-0) - (e^{-2\pi}-1)\right]$$

$$= \frac{1-e^{-2\pi}}{\pi(1+n^2)}$$

$$\boxed{a_n = \left(\frac{1}{n^2+1}\right)\left(\frac{1-e^{-2\pi}}{\pi}\right)}$$

$$b_n = \frac{1}{\pi}\int_a^b f(x).\sin nx\,dx = \frac{1}{\pi}\int_0^{2\pi} e^{-x}.\sin nx\,dx$$

Use formula,

$$\int e^{ax}.\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$b_n = \frac{1}{\pi}\left[\frac{e^{-x}}{(1+n^2)}(-\sin nx - n\cos nx)\right]_0^{2\pi}$$

$$= \frac{-1}{\pi(1+n^2)}\left[e^{-x}\sin nx + n.e^{-x}\cos nx\right]_0^{2\pi}$$

Div:-A Present 8to9 26/05/23 | 28, 23, 34, 02, 85, 39, ... 19, 05.

$$= \frac{-1}{\pi(1+n^2)}\left[(0-0)+(ne^{-2\pi}-n)\right]$$

$$= \frac{n(1-e^{-2\pi})}{\pi(n^2+1)}$$

$$\boxed{b_n = \frac{n}{n^2+1}\left(\frac{1-e^{-2\pi}}{\pi}\right)}$$

eq^n (1) becomes,

$$e^{-x} = \left(\frac{1-e^{-2\pi}}{2\pi}\right) + \sum_{n=1}^{\infty}\left(\frac{1-e^{-2\pi}}{\pi}\right)\left(\frac{1}{n^2+1}\right)\cos nx$$

$$+ \sum_{n=1}^{\infty}\left(\frac{1-e^{-2\pi}}{\pi}\right)\left(\frac{n}{n^2+1}\right)\sin nx$$

$$= \left(\frac{1-e^{-2\pi}}{\pi}\right)\left\{\frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{1}{n^2+1}\right)\cos nx + \sum_{n=1}^{\infty}\left(\frac{n}{n^2+1}\right)\sin nx\right\}$$

$$= \left(\frac{1-e^{-2\pi}}{\pi}\right)\left\{\frac{1}{2} + \left(\frac{1}{2}\cos x + \frac{1}{5}\cos 2x + \frac{1}{10}\cos 3x + \dots\right)\right.$$

$$\left. + \left(\frac{1}{2}\sin x + \frac{2}{5}\sin 2x + \frac{3}{10}\sin 3x + \dots\right)\right\}$$

E.X. Find a fourier series to represent $(x-x^2)$ from $-\pi < x < \pi$.

$\longrightarrow$ Here,

$f(x) = x - x^2$ and $b-a = 2\pi$

The fourier series for $f(x) = x - x^2$ is

$$x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{--- (1)}$$

$$a_0 = \frac{1}{\pi}\int_a^b f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\,dx = \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{(-\pi)^2}{2} - \frac{\pi^3}{3} + \frac{(-\pi)^3}{3}\right]$$

8 to 9 Present 29/05/23 | 45, 46, 47, 48, 50, 51, ... 19, 21, 22, 24, 29, 31, 32, 34, 35, 36, 37, 38, 41, 43, ... 56, 57, 58, 61, 63, 64, 65, 66, 67, 69

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{\pi^2}{2} - \frac{\pi^3}{3} - \frac{\pi^3}{3}\right]$$

$$= -\frac{2\pi^3}{3\pi}$$

$$\boxed{a_0 = -\frac{2}{3}\pi^2}$$

$$a_n = \frac{1}{\pi}\int_a^b f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\frac{\sin nx}{n} - (1-2x)\cdot\left(-\frac{\cos nx}{n^2}\right) + (-2)\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(x-x^2)\frac{\sin nx}{n} + (1-2x)\frac{\cos nx}{n^2} + \frac{2\sin nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\cancel{(\pi-\pi^2)\frac{\sin \pi}{n}} + -\cancel{(-\pi-\pi^2)\frac{\sin n(-\pi)}{n}} + (1-2\pi)\frac{\cos n\pi}{n^2} - \frac{(1+2\pi)\cos n(-\pi)}{n^2} + \cancel{\frac{2\sin n\pi}{n^3}} - \cancel{\frac{2\sin n(-\pi)}{n^3}}\right]$$

$$= \frac{1}{\pi}\left[\frac{(1-2\pi)(-1)^n}{n^2} - \frac{(1+2\pi)(-1)^n}{n^2}\right]$$

$$= \frac{1}{n^2\pi}\left[(-1)^n - 2\pi(-1)^n - (-1)^n - 2\pi(-1)^n\right]$$

$$\boxed{a_n = \frac{-4(-1)^n}{n^2}}$$

$$b_n = \frac{1}{\pi}\int_a^b f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(-\frac{\cos nx}{n}\right) - (1-2x)\left(-\frac{\sin nx}{n^2}\right) + (-2)\frac{\cos nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(x^2-x)\frac{\cos nx}{n} + (1-2x)\frac{\sin nx}{n^2} - 2\frac{\cos nx}{n^3}\right]_{-\pi}^{\pi}$$

Div:-B Present 29/05/23 11 to 12 12:30 to 1:30

24, 34, 20, 16, 17, 28, 15, 33, 26, 13, 14, 21, 03, 02, ...

23, 22, 39, 19, 01, 29. 39, 22, 01, 19, 29.

→ 21, 24, 34, 20, 16, 17, 15, 28, 26, 33, 10, 11, 09, 03, 02, 14, 13, 07, 31, 23, 08,

$$= \frac{1}{\pi}\left[(\pi^2-\pi)\frac{\cos n\pi}{n} - (\pi^2+\pi)\frac{\cos n(-\pi)}{n} + (1-2\pi)\frac{\sin n\pi}{n^2} - (1+2\pi)\frac{\sin n(-\pi)}{n^2} - 2\frac{\cos n\pi}{n^3} + 2\frac{\cos n(-\pi)}{n^3}\right]$$

$$= \frac{1}{\pi}\left[(\pi^2-\pi)\frac{(-1)^n}{n} - (\pi^2+\pi)\frac{(-1)^n}{n}\right]$$

$$= \frac{(-1)^n}{n\pi}\left[\pi^2 - \pi - \pi^2 - \pi\right]$$

$$= \frac{(-1)^n}{n\pi}(-2\pi)$$

$$\boxed{b_n = \frac{-2(-1)^n}{n}}$$

From ①,

$$x - x^2 = \left(\frac{-2\pi^2}{3}\right)\cdot\frac{1}{2} + \sum_{n=1}^{\infty}\frac{(-4)(-1)^n}{n^2}\cos nx + \sum_{n=1}^{\infty}\frac{(-2)(-1)^n}{n}\sin nx$$

$$= -\frac{\pi^2}{3} - 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx - 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nx$$

$$= \frac{-\pi^2}{3} + 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \cdots\right) + 2\left(\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right)$$

30/05/23 Present 8 to 9 Div:-C → 06, 66, 18, 02, 61, 55, 05, 46, 68, 36, 24, 41, 56, 35, 64, 14, 46, 34, 48, 52, 07, 21, 37, 67, 45, 47, 04, 69, 29, 33, 12, 57, 50, 30, 65, 51, 43, 09, 32,

Div C: C_3 25/05/23 Present 2:30 to 3:30 — 65, 51, 58, 59, 47, 49, 60, 64, 68, 55, 48, 61, 66, 56, 63, 53.

Div:-C C_2 1:30 to 2:30 Present 26/05/23 — 45, 29, 33, 38, 32, 37, 41, 46, 40, 31, 34, 28, 25, 27, 36, Sent 30/05/23 11 to 12

01, 27, 36, 15, 19, 05, 13, 17, 24, 25, 26, 07, 30, 03 ... 31, 42, 34, 35, 04 ...

U.4.2

THE MET LEAGUE OF COLLEGES
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Institute of Technology - Polytechnic - Diploma/B.Tech
CLASS TEST - I / II / Mid Sem Exam Date :

Name : Roll No :

Course Title : Class : Sem :

Division :

Main Answer book	Supplement	Total	Question No.	1						2				3				Total Marks
			Max. Marks															
Supervisor's Sign.				a	b	c	d	e	f	a	b	c	d	a	b	c	d	
			Marks Obtained															

E.X. Find the Fourier Series for $f(x)=\sqrt{1-\cos x}$ in the interval $0<x<2\pi$ & Deduce that $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}=\frac{1}{2}$.

→ Here,

$$f(x)=\sqrt{1-\cos x}=\sqrt{2\sin^2\frac{x}{2}}=\sqrt{2}\sin\frac{x}{2}$$

& $b-a=2\pi$

∴ The fourier series for $f(x)=\sqrt{1-\cos x}$ is,

$$\sqrt{1-\cos x}=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx \quad \text{——— (1)}$$

$$a_0=\frac{1}{\pi}\int_a^b f(x)\,dx=\frac{1}{\pi}\int_0^{2\pi}\sqrt{2}\sin\frac{x}{2}\,dx$$

$$=\frac{\sqrt{2}}{\pi}\left[-2\cdot\cos\frac{x}{2}\right]_0^{2\pi}=\frac{2\sqrt{2}}{\pi}\left[-\cos\pi+\cos 0\right]$$

$$\boxed{a_0=\frac{4\sqrt{2}}{\pi}}$$

$$a_n=\frac{1}{\pi}\int_0^{2\pi}f(x)\cos nx\,dx=\frac{1}{\pi}\int_0^{2\pi}\left(\sqrt{2}\sin\frac{x}{2}\right)\cos nx\,dx$$

$$=\frac{\sqrt{2}}{2\pi}\int_0^{2\pi}2\sin\frac{x}{2}\cdot\cos nx\,dx$$

$$=\frac{1}{\sqrt{2}\pi}\int_0^{2\pi}\left[\sin\left(n+\tfrac{1}{2}\right)x-\sin\left(n-\tfrac{1}{2}\right)x\right]dx$$

Div:-A 12130401:30 30/05/23 Present

02, 01, 37, 22, 20, 38, 40

$$= \frac{1}{\sqrt{2}\pi}\left[-\frac{\cos\left(\frac{2n+1}{2}\right)x}{\left(\frac{2n+1}{2}\right)} + \frac{\cos\left(\frac{2n-1}{2}\right)x}{\left(\frac{2n-1}{2}\right)}\right]_0^{2\pi}$$

$$= \cancel{\frac{2}{\sqrt{2}\pi}\left[\left\{-\frac{\cos(2n+1)\pi}{(2n+1)}\right.\right.}$$

$$= \frac{1}{\pi\sqrt{2}}\left[\left(\frac{-2}{2n+1}\right)\cos\left(\frac{2n+1}{2}\right)x + \left(\frac{2}{2n-1}\right)\cos\left(\frac{2n-1}{2}\right)x\right]_0^{2\pi}$$

$$= \frac{2}{\pi\sqrt{2}}\left\{\left[\left(\frac{-1}{2n+1}\right)\cos\left(\frac{2n+1}{2}\right)2\pi + \left(\frac{1}{2n-1}\right)\cos\left(\frac{2n-1}{2}\right)2\pi\right]\right.$$
$$\left. - \left[\left(\frac{-1}{2n+1}\right)\cos\left(\frac{2n+1}{2}\right)(0) + \left(\frac{1}{2n-1}\right)\cos\left(\frac{2n-1}{2}\right)(0)\right]\right\}$$

$$= \frac{2}{\pi\sqrt{2}}\left\{\left[\left(\frac{-1}{2n+1}\right)\cos(2n+1)\pi + \left(\frac{2}{2n-1}\right)\cos(2n-1)\cdot\pi\right]\right.$$
$$\left. - \left[\left(\frac{-1}{2n+1}\right)\cos(0) + \left(\frac{1}{2n-1}\right)\cos(0)\right]\right\}$$

$$= \frac{2}{\pi\sqrt{2}}\left\{\left[\left(\frac{-1}{2n+1}\right)(-1) + \left(\frac{1}{2n-1}\right)(-1)\right] - \left[\left(\frac{-1}{2n+1}\right) + \left(\frac{1}{2n-1}\right)\right]\right\}$$

$$= \frac{2}{\pi\sqrt{2}}\left[\frac{1}{2n+1} - \frac{1}{2n-1} + \frac{1}{2n+1} - \frac{1}{2n-1}\right]$$

$$= \frac{\sqrt{2}}{\pi}\left[\frac{2}{2n+1} - \frac{2}{2n-1}\right]$$

$$\boxed{a_n = \frac{-4\sqrt{2}}{\pi(4n^2-1)}}$$

$$b_n = \frac{1}{\pi}\int_a^b f(x)\cdot\sin nx\,dx = \frac{1}{\pi}\int_0^{2\pi}\sqrt{2}\sin\frac{x}{2}\sin nx\,dx$$

Div:- 30/05 Pres 21, 20

#

EX

D'IV-B
30/05/23
Present
2:30 to 3:30

1,3,4,5,6,7,8,11,15,17,19,20,21,22,23,24,25,28,29,
30,31,32,33,34,39,42.

$$= \frac{\sqrt{2}}{2\pi}\int_0^{2\pi}\left(2\sin\frac{x}{2}\cdot\sin nx\right)dx$$

$$= \frac{1}{\sqrt{2}\,\pi}\int_0^{2\pi}\left[\cos\left(n-\frac{1}{2}\right)x - \cos\left(n+\frac{1}{2}\right)x\right]dx$$

$$= \frac{1}{\sqrt{2}\,\pi}\left[\frac{\sin\left(\frac{2n-1}{2}\right)x}{\left(\frac{2n-1}{2}\right)} - \frac{\sin\left(\frac{2n+1}{2}\right)x}{\left(\frac{2n+1}{2}\right)}\right]_0^{2\pi}$$

$$\boxed{b_n = 0}$$

$\therefore$ eq^n (1) becomes,

$$\sqrt{1-\cos x} = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{4n^2-1}\right)\cos nx \quad \text{--- (2)}$$

putting $x=0$ in (2),

$$0 = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$$

$$\therefore \sum_{n=1}^{\infty}\frac{1}{4n^2-1} = \frac{1}{2}$$

Functions having points of discontinuity

EX. Find the fourier series expansion for

$$f(x), \text{ if } f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

→ Here, $b-a = 2\pi$

$\therefore$ The fourier series is given by

Div:-A
8 to 9
Present
31/05/23

28, 02, ... 04, 09, 18, 37, 39, 12, 34, 17,
33, 27, 19, 36, 30, 03, ... 24, 25.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{--- (1)}$$

$$a_0 = \frac{1}{\pi}\int_a^b f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\,dx + \int_0^{\pi} x\,dx\right]$$

$$= \frac{1}{\pi}\left\{\left[(-\pi)\,x\right]_{-\pi}^{0} + \left[\frac{x^2}{2}\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left[(-\pi)(0+\pi) + \left(\frac{\pi^2}{2} - 0\right)\right]\right\}$$

$$= \frac{1}{\pi}\left(-\pi^2 + \frac{\pi^2}{2}\right)$$

$$\boxed{a_0 = -\frac{\pi}{2}}$$

$$a_n = \frac{1}{\pi}\int_a^b f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\cos nx\,dx + \int_0^{\pi} x.\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left\{\left[(-\pi)\frac{\sin nx}{n}\right]_{-\pi}^{0} + \left[x\left(\frac{\sin nx}{n}\right) - (1)\left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{-\pi}{n}\right)\left(\sin \cancel{0} - \sin \cancel{n(-\pi)}\right)\right] + \right.$$

$$\left.\left[\left(\pi\left(\frac{\sin n\pi}{n}\right) + \left(\frac{\cos n\pi}{n^2}\right)\right) - \left(0 + \frac{\cos 0}{n^2}\right)\right]\right\}$$

$$= \frac{1}{\pi}\left\{\cancel{\left(\frac{-\pi}{n}\right)} + \pi.\frac{\sin n\pi}{n} + \frac{\cos n\pi}{n^2} - \frac{\cos 0}{n^2}\right\}$$

$$= \frac{1}{\pi}\left\{\cancel{\left(\frac{-\pi}{n}\right)} + \frac{\cos n\pi}{n^2} - \frac{\cos 0}{n^2}\right\}$$

Div:-C 31/05/23 11 to 12 Present | 01, 02, 03, 05, 06, 07, 08, 09, 10, 11, 12, 14, 15, 16, 17, 18, 19, ... 45, 46, 47, 26, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 43, 44, 66, 67, 68, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65,

$$= \frac{1}{\pi}\left\{ \cancel{\frac{-\pi}{n}} + \frac{(-1)^n}{n^2} - \frac{1}{n^2} \right\}$$

$$= \cancel{\frac{\pi}{n}} \cdot \frac{1}{\pi}\left[\frac{(-1)^n - 1}{n^2}\right]$$

$$\therefore \boxed{a_n = \frac{(-1)^n - 1}{n^2\pi}}$$

$$b_n = \frac{1}{\pi}\int_a^b f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left\{\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_0^{\pi} f(x)\sin nx\,dx\right]\right\}$$

$$\cancel{= \frac{1}{\pi}\left\{\left[\int_{-\pi}^{0}(-\pi)\left(-\frac{\cos nx}{n}\right)dx\right] + \left[\int_0^{\pi} x\sin nx\,dx\right]\right\}}$$

$$= \frac{1}{\pi}\left\{(-\pi)\left(-\frac{\cos nx}{n}\right)_{-\pi}^{0} + \left[x\cdot\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left[(\pi)\left\{\frac{\cos 0}{n} - \frac{\cos n(-\pi)}{n}\right\} - \frac{\pi\cos n\pi}{n} + 0 + \frac{\sin n\pi}{n^2} - \frac{\sin 0}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\pi\left(\frac{1}{n} - \frac{\cos n\pi}{n}\right) - \frac{\pi\cos n\pi}{n}\right]$$

$$= \cancel{\frac{1}{\pi}}\left[\cancel{\pi}\right. = \frac{1}{\pi}\cdot\frac{\pi}{n}\left[1 - 2\cos n\pi\right]$$

$$\therefore \boxed{b_n = \frac{1}{n}\left[1 - 2(-1)^n\right]}$$

Div:- C
Present
12:30 to 1:30
31/05/23

01, 02, ... 06, 07, 08, 09, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 29, 30, 31, 32, 33, 34, 36, 37, 38, 40, 41, 43, 45, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 63, 64, 65, 66, 67, 68.

Div: C
Present
31/05
1:30 to

$\therefore$ eq^n (1) becomes,

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \left\{ \frac{(-1)^n - 1}{n^2 \pi} \right\} \cos nx + \sum_{n=1}^{\infty} \left\{ \frac{1 - 2(-1)^n}{n} \right\} \sin nx$$

E.X. If $f(x) = \begin{cases} 0 , & -\pi \le x \le 0 \\ \sin x , & 0 \le x \le \pi \end{cases}$; prove that

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos nx}{n^2 - 1}$$ & hence

show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} ------ \infty = \frac{1}{4}(\pi - 2)$

$\rightarrow$ Here, $b - a = 2\pi$

$\therefore$ The Fourier Series for function $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{--- (1)}$$

$$a_0 = \frac{1}{\pi} \int_a^b f(x)\, dx = \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 . dx + \int_0^{\pi} \sin x\, dx \right]$$

$$= \cancel{\frac{1}{2\pi} \int_0^{\pi} 2} \qquad = \frac{1}{\pi} \left[(-\cos x) \right]_0^{\pi}$$

$$= \frac{-1}{\pi} \left(\cos \pi - \cos 0 \right)$$

$$= \frac{-1}{\pi} (-1 - 1)$$

$$\boxed{a_0 = \frac{2}{\pi}}$$

$$a_n = \frac{1}{\pi} \int_a^b f(x) \cos nx\, dx = \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 . \cos nx\, dx + \int_0^{\pi} \sin x . \cos nx \right.$$

$$= \frac{1}{2\pi} \int_0^{\pi} 2 \cos nx . \sin nx\, dx$$

Div:-B Present 31/05/23 1:30 to 2:30 | 1, 2, , 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28, 29, 30, 31, 32, 33, 34, 39,

$$= \frac{1}{2\pi} \int_0^{\pi} \left\{ \sin(n+1)x - \sin(n-1)x \right\} dx$$

$$= \frac{1}{2\pi} \left[\frac{-\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1} \right]_0^{\pi}$$

$2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

$$= \frac{1}{2\pi} \left[\frac{\cos(n-1)\pi}{(n-1)} - \frac{\cos 0}{n-1} - \frac{\cos(n+1)\pi}{n+1} + \frac{\cos 0}{n+1} \right]$$

$$= \frac{1}{2\pi} \left[\frac{(-1)^{n-1} - 1}{n-1} + \frac{1 - (-1)^{n+1}}{n+1} \right] \quad (n \neq 1) \text{ i.e. } n = 2 \text{ to } \infty$$

when n is odd, $a_n = 0$

when n is even, $a_n = \frac{1}{2\pi} \left[\frac{-2}{n-1} + \frac{2}{n+1} \right]$

$$= \frac{1}{2\pi} \left[\frac{-2n-2+2n-2}{n^2-1} \right]$$

$$\int_a^b = \int_a^0 + \int_0^b$$

$$= \frac{-4}{2\pi(n^2-1)}$$

$$\int \sin ax = -\frac{\cos ax}{a}$$

$$\boxed{a_n = \frac{-2}{(n^2-1)\pi}}$$

$$b_n = \frac{1}{\pi} \int_a^b f(x) \cdot \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 \cdot \sin nx \, dx + \int_0^{\pi} \sin x \cdot \sin nx \, dx \right]$$

$$= \frac{1}{2\pi} \int_0^{\pi} 2 \sin nx \cdot \sin x \, dx$$

$2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

$$= \frac{1}{2\pi} \int_0^{\pi} \left\{ \cos(n-1)x - \cos(n+1)x \right\} dx$$

$$= \frac{1}{2\pi} \left[\frac{\sin(n-1)x}{n-1} - \frac{\sin(n+1)x}{n+1} \right]_0^{\pi} = 0 \quad (n \neq 1)$$

$$b_1 = \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 \cdot \sin x \, dx + \int_0^{\pi} \sin^2 x \, dx \right]$$

$\because \cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

Div:-B Present 1/06/23 8 to 9 | 01, 02, 04, 05, 07, 08, 10, ... 36, 39,

$$= \frac{1}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2x}{2}\right)dx = \frac{1}{2\pi}\left[\int_0^{\pi}dx - \int_0^{\pi}\cos 2x\,dx\right]$$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$$= \frac{1}{2\pi}\left[(x)_0^{\pi} - \left(\frac{\sin 2x}{2}\right)_0^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\pi - \frac{1}{2}(\sin 2\pi - \sin 0)\right]$$

$$\therefore \boxed{b_1 = \frac{1}{2}}$$

{ rough work. $a_0 = 2/\pi$, $a_n = \frac{-2}{(n^2-1)\pi}$ $(n \neq 1)$, $a_1 = 0$, $b_n = 0$, $b_1 = 1/2$ }

$b_1, b_2, b_3 \ldots b_n$

$\therefore$ eqn (1),

$$f(x) = \frac{1}{\pi} - \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{\cos nx}{n^2-1} + \frac{1}{2}\sin x$$

$$= \frac{1}{\pi} - \frac{2}{\pi}\left(\frac{\cos 2x}{2^2-1} + \frac{\cos 4x}{4^2-1} + \frac{\cos 6x}{6^2-1} + \cdots\right) + \frac{1}{2}\sin x$$

put $x = \pi/2$.

$$\sin \pi/2 = \frac{1}{\pi} - \frac{2}{\pi}\left(\frac{\cos \pi}{3} + \frac{\cos 2\pi}{15} + \frac{\cos 3\pi}{35} + \cdots\right) + \frac{1}{2}$$

$$1 - \frac{1}{2} = \frac{1}{\pi} - \frac{2}{\pi}\left(-\frac{1}{3} + \frac{1}{15} - \frac{1}{35} + \cdots\right)$$

$$\frac{1}{2} - \frac{1}{\pi} = \frac{2}{\pi}\left(\frac{1}{3} - \frac{1}{15} + \frac{1}{35} - \cdots\right) = \frac{\pi - 2}{2\pi}$$

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots = \frac{1}{4}(\pi - 2)$$

Hence proved

Div:-C1 Present 01/06/23 9 to 10 | 22, 09, 20, 03, 02, 06, 14, 21, 07, 48, 15.

Div:-A Present 01/06/23 1:30 to 1:30 | 1, 2, 28, 12, 8, 42, 35, 34, 40, 4, 18, 9, 38, 39, 37, 36, 27, 33, 30, 03, 26, 07, 11, 24, 25, 19, 5, 15, 22.

Present Div C3 → 65, 51, 58, 64, 47, 48, 63, 37, 45, 55, 56, 66, 52, 68, 03, Div-C Present 1:30 to 2:30

41, 39, 40, 30, 45, 37, 46, 34, 26, 20, 36.

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Institute of Technology - Polytechnic - Diploma/B.Tech
CLASS TEST - I / II / Mid Sem Exam Date :

Name : ________ Roll No : ________

Course Title : ________ Class : ________ Sem : ________

Division :

Main Answer book	Supplement	Total	Question No.	1						2				3				Total Marks
			Max. Marks															
				a	b	c	d	e	f	a	b	c	d	a	b	c	d	
Supervisor's Sign.			Marks Obtained															

Change of Interval

E.X. Find the Fourier series expansion of $f(x) = 2x - x^2$ in the interval $(0,3)$.

→ Here,

$f(x) = 2x - x^2$ and $b - a = 3$

∴ The Fourier Series of $f(x)$ is given by

$$f(x) = 2x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{3}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi x}{3}\right) \quad \text{--- (1)}$$

$$a_0 = \frac{2}{3}\int_0^3 (2x - x^2)\,dx = \frac{2}{3}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \frac{2}{3}\left[x^2 - \frac{x^3}{3}\right]_0^3 = \frac{2}{3}\left[\left(3^2 - \frac{3^3}{3}\right) - (0)\right]$$

$$= \frac{2}{3}\left(9 - \frac{27}{3}\right) = 0$$

$$\therefore \boxed{a_n = 0}$$

$$a_n = \frac{2}{3}\int_0^3 \underset{I}{(2x - x^2)} \underset{II}{\cos\left(\frac{2n\pi x}{3}\right)}dx \qquad \text{ILATE}$$

$$= \frac{2}{3}\left[(2x - x^2)\frac{\sin(2n\pi x/3)}{2n\pi/3} - (2 - 2x)\left\{\frac{-\cos(2n\pi x/3)}{4n^2\pi^2/9}\right\} + (0 - 2)\left\{\frac{-\sin(2n\pi x/3)}{8n^3\pi^3/27}\right\}\right]_0^3$$

Div:-A 8 to 9 Roll Absent 2/06/23

01, 07, 08, 10, ... 40, 41, ...

Div:- 10 to ... Absent ...

$$= \frac{2}{3}\left[\frac{3(2x-x^2)}{2n\pi}\cdot\sin\left(\frac{2n\pi x}{3}\right) + \frac{9(2-2x)}{4n^2\pi^2}\cos\left(\frac{2n\pi x}{3}\right) + \frac{54}{8n^3\pi^3}\cdot\sin\left(\frac{2n\pi x}{3}\right)\right]_0^3$$

$$= \frac{2}{3}\left[\frac{3(-3)}{2n\pi}\sin 2n\pi - 0 + \frac{(-36)}{4n^2\pi^2}\cdot\cos 2n\pi - \frac{18}{4n^2\pi^2}\cos 0 + \frac{54}{8n^3\pi^3}\sin 2n\pi - 0\right]$$

Upper limit

lower limit

$$= \frac{2}{3}\left[\frac{-9}{n^2\pi^2}(1) - \frac{9}{2n^2\pi^2}\right]$$

$$= -\frac{2}{3}\left(\frac{9}{n^2\pi^2}\right)\left(1+\frac{1}{2}\right)$$

$$= -\frac{2}{3}\left(\frac{9}{n^2\pi^2}\right)\left(\frac{3}{2}\right)$$

Imp #

$$\boxed{a_n = \frac{-9}{n^2\pi^2}}$$

$$b_n = \frac{2}{3}\int_0^3 (2x-x^2)\sin\left(\frac{2n\pi x}{3}\right)dx$$

$$= \frac{2}{3}\left[(2x-x^2)\left(\frac{-\cos(2n\pi x/3)}{2n\pi/3}\right) - (2-2x)\left(\frac{-\sin(2n\pi x/3)}{4n^2\pi^2/9}\right) + (-2)\left(\frac{\cos(2n\pi x/3)}{8n^3\pi^3/27}\right)\right]_0^3$$

$$= \frac{2}{3}\left[\frac{3(x^2-2x)}{2n\pi}\cos(2n\pi x/3) + \frac{9(2-2x)}{4n^2\pi^2}\sin\left(2n\pi x/3\right) - \frac{54}{8n^3\pi^3}\cos\left(2n\pi x/3\right)\right]_0^3$$

$$= \frac{2}{3}\left[\frac{9}{2n\pi}\cos 2n\pi - 0 + \frac{(-36)}{4n^2\pi^2}\sin 2n\pi - \frac{18}{4n^2\pi^2}\sin 0 - \frac{27}{4n^3\pi^3}\cdot\cos 2n\pi + \frac{27}{4n^3\pi^3}\cos 0\right]$$

Div:- ... 10 to 11 Absent 2/06/23 | 61, 62, 68, 69.

$$= \frac{2}{3}\left[\frac{9}{2n\pi}\right]$$

$$\boxed{b_n = \frac{3}{n\pi}}$$

$\therefore$ From (1),

$$2x - x^2 = \frac{-9}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(\frac{2n\pi x}{3}\right) + \frac{3}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{2n\pi x}{3}\right)$$

which is the required fourier series.

Imp

Even and odd functions

Interval $(-a, a)$

e.g. $(-\pi, \pi)$ or $(-1, 1)$ or $(-2, 2)$ or $(-l, l)$

Then check for even or odd function

For Even function

Put $x = -x$

If $f(-x) = f(x)$

then function is even. $(b_n = 0)$

The fourier series of an even function is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{b-a}\right)$$

For odd function

Put $x = -x$

If $f(-x) = -f(x)$

then function is odd. $(a_0 = 0, a_n = 0)$

The fourier series of an odd ~~&~~ function is,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi x}{b-a}\right)$$

To solve a_0, a_n and b_n, we adopt their Methodology,

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

E.X. Find the fourier Series of $f(x) = e^{-|x|}$ in $(-\pi, \pi)$.

→ Here,

$f(x) = e^{-|x|}$

Put $x = -x$;

$f(-x) = e^{-|x|} = e^{-|x|} = f(x)$

Hence, $f(x)$ is an even function.

$\Rightarrow b_n = 0$

Now,

$$f(x) = e^{-(-x)} = e^{x} \; ; \; -\pi < x < 0$$
$$= e^{-(x)} = e^{-x} \; ; \; 0 < x < \pi$$

∴ The fourier Series of an even function is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi x}{b-a}$$

Here, $b-a = 2\pi$

$$\therefore e^{-|x|} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{—— (1)}$$

$$a_0 = \frac{2}{b-a}\int_a^b f(x)\,dx = \frac{2}{2\pi}\int_{-\pi}^{\pi} e^{-|x|}\,dx$$

$$= \frac{1}{\pi}\, 2\int_0^{\pi} e^{-x}\,dx = \frac{2}{\pi}\left[\frac{e^{-x}}{-1}\right]_0^{\pi}$$

$$= \frac{-2}{\pi}\left(e^{-\pi} - e^{0}\right).$$

$$\boxed{a_0 = \frac{2}{\pi}\left(1 - e^{-\pi}\right)}$$

$$a_n = \frac{2}{b-a}\int_a^b f(x)\cos\left(\frac{2n\pi x}{b-a}\right)dx$$

$$= \frac{2}{2\pi}\int_{-\pi}^{\pi} e^{-|x|}\cos nx\,dx$$

Div:- B :- 6, 12, 15, 16, 22, 25, 27, 35, 37, 40,
absent

Div:- A :- 1, 3, 5, 7, 8, 10, 11, 12, 21, 22, 23, 26, 29, 30, 32, 36, 37, 40, 41, 42
02/06/23
12:30 - 01:30

$$= \frac{1}{\pi} \cdot 2 \int_0^{\pi} e^{-x} \cos nx \, dx$$

Use formula,

$$\int e^{ax} \cdot \cos bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$$

$$= \frac{2}{\pi} \left[\frac{e^{-x}}{(1+n^2)} (-\cos nx + n \sin nx) \right]_0^{\pi}$$

$$= \frac{2}{(1+n^2)\pi} \left[e^{-\pi} (-\cos n\pi + n \sin n\pi) - e^{0}(-\cos 0 + n \sin 0) \right]$$

$$= \frac{2}{(1+n^2)\pi} \left[-(-1)^n \cdot e^{-\pi} + 1 \right]$$

$$a_n = \frac{2}{(1+n^2)\pi} \left[1 - (-1)^n e^{-\pi} \right]$$

$\therefore$ From eq^n (1),

$$e^{-|x|} = \left(\frac{1-e^{-\pi}}{\pi} \right) + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{1+n^2} \left[1 - (-1)^n e^{-\pi} \right] \cos nx$$

which is the required fourier series.

E.X. Obtain a fourier series for the function

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$$

$\rightarrow$ Put $x = -x$

$f(-x) = 1 - \frac{2x}{\pi}$ in $(-\pi, 0)$ $= f(x)$ in $(0, \pi)$

Also, $f(-x) = 1 + \frac{2x}{\pi}$ in $(0, \pi) = f(x)$ in $(-\pi, 0)$

U.4.4

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CLASS TEST - I / II / Mid Sem Exam Date :

Name : ______ Roll No : ______

Course Title : ______ Class : ______ Sem : ______

Division :

Main Answer book	Supplement	Total	Question No.	1						2				3				Total Marks
			Max. Marks															
				a	b	c	d	e	f	a	b	c	d	a	b	c	d	
Supervisor's Sign.			Marks Obtained															

Half - Range Series

① Cosine Series OR Half Range Cosine Series

The Fourier half range cosine series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\cancel{2}n\pi x}{b-a}\right)$$

where,

$$a_0 = \frac{2}{b-a}\int_a^b f(x)\,dx$$

$$a_n = \frac{2}{b-a}\int_a^b f(x) \cos\left(\frac{n\pi x}{b-a}\right)dx$$

② Sine Series OR Half Range Sine Series

The Fourier half range Sine series is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{b-a}\right)$$

where,

$$b_n = \frac{2}{b-a}\int_a^b f(x) \sin\left(\frac{n\pi x}{b-a}\right)dx$$

E.X. Find the Fourier half range cosine series of the function

$$f(t) = \begin{cases} 2t & ; \ 0<t<1 \\ 2(2-t) & ; \ 1<t<2 \end{cases}$$

→ The reqd fourier half range cosine series is given by,

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{b-a}\right)$$

Here, $b-a = 2-0 = 2$

$$\therefore f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{2}\right) \quad \text{---} \ (1)$$

$$a_0 = \frac{2}{b-a}\int_a^b f(t)\,dt$$

$$= \frac{2}{2}\left[\int_0^1 2t\,dt + \int_1^2 2(2-t)\,dt\right]$$

$$= 2\int_0^1 t\,dt + 2\int_1^2 (2-t)\,dt$$

$$= 2\left[\frac{t^2}{2}\right]_0^1 + 2\left[2t - \frac{t^2}{2}\right]_1^2$$

$$= 2\left(\frac{1}{2}\right) + 2\left[\left(2(2) - \frac{2^2}{2}\right) - \left(2(1) - \frac{1}{2}\right)\right]$$

$$= 1 + 2\left[\left(4 - \frac{4}{2}\right) - \left(2 - \frac{1}{2}\right)\right]$$

$$= 1 + 2\left[(4-2) - \left(\frac{4-1}{2}\right)\right]$$

$$= 1 + 2(2) - \frac{2(4-1)}{2}$$

$$= 1 + 4 - 4 + 1$$

$$\boxed{a_0 = 2}$$

$$a_n = \frac{2}{b-a}\int_a^b f(t)\cos\left(\frac{n\pi t}{b-a}\right)dt$$

$$= \frac{2}{2}\left[\int_0^1 2t\cos\left(\frac{n\pi t}{2}\right)dt + \int_1^2 (4-2t)\cos\left(\frac{n\pi t}{2}\right)dt\right]$$

$$= 2\left[t\cdot\frac{\sin(n\pi t/2)}{n\pi/2} - (1)\left(\frac{-\cos(n\pi t/2)}{n^2\pi^2/4}\right)\right]_0^1$$

$$+\left[(4-2t)\left(\frac{\sin(n\pi t/2)}{n\pi/2}\right) - (-2)\left(\frac{-\cos(n\pi t/2)}{n^2\pi^2/4}\right)\right]_1^2$$

$$= 2\left[\frac{2t}{n\pi}\sin\left(\frac{n\pi t}{2}\right) + \frac{4}{n^2\pi^2}\cos\left(\frac{n\pi t}{2}\right)\right]_0^1$$

$$+\left[\frac{2(4-2t)}{n\pi}\cdot\sin\left(\frac{n\pi t}{2}\right) - \frac{8}{n^2\pi^2}\cos\left(\frac{n\pi t}{2}\right)\right]_1^2$$

$$= 2\left[\frac{2}{n\pi}\sin\frac{n\pi}{2} - 0 + \frac{4}{n^2\pi^2}\cdot\cos\frac{n\pi}{2} - \frac{4}{n^2\pi^2}\cos 0\right]$$

$$+ 0 - \frac{4}{n\pi}\sin\frac{n\pi}{2} - \frac{8}{n^2\pi^2}\cos n\pi + \frac{8}{n^2\pi^2}\cos\frac{n\pi}{2}$$

$$= \cancel{\frac{4}{n\pi}\sin\frac{n\pi}{2}} + \frac{8}{n^2\pi^2}\cos\frac{n\pi}{2} - \frac{8}{n^2\pi^2} - \cancel{\frac{4}{n\pi}\sin\frac{n\pi}{2}}$$

$$-\frac{8}{n^2\pi^2}\cos n\pi + \frac{8}{n^2\pi^2}\cos\frac{n\pi}{2}$$

$$a_n = \frac{8}{n^2\pi^2}\left[2\cos\frac{n\pi}{2} - 1 - (-1)^n\right]$$

From eq^n (1),

$$f(t) = 1 + \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[2\cos\frac{n\pi}{2} - 1 - (-1)^n\right]\cos\frac{n\pi t}{2}$$